Enhanced Regular Corecursion for Data Streams

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Introduction
As we venture deeper into the Internet of Things (IoT) era, stream processing is becoming increasingly important. With our recent work, we propose a stream calculus to lay the foundations of a tool for testing of IoT systems and real time analysis of unbound data series.

Main Objectives
• Develop a calculus to define and manipulate infinite streams
• Provide procedures to check well-definedness and equality of streams
• Achieve a good compromise between expressive power and decidability

State of the Art
Two complementary approaches to manipulate streams:

Lazy Evaluation
Streams defined by arbitrary functions and inspected as much as needed.
Pros
• Widely known and well-established solution for stream processing
• Supports both regular (cyclic) and non-regular streams
Cons
• Operations that need to inspect the whole stream cannot be computed
• Allows the definition of undefined streams due to high expressive power

Regular Corecursion
Streams are represented by finitary equational systems. Non-termination of recursive stream functions is avoided by keeping track of already processed calls.
Pros
• The entire stream can be inspected because it is finitely represented by an equational system
Cons
• Fails to model non-regular streams

Our Solution
• Enhances regular corecursion
  – Not only constructors are allowed in equations defining streams
  – Besides regular streams, supports also a subset of non-regular streams
• Provides procedures to check well-definedness and equality of streams

Examples
Simple cyclic streams
\[ \text{repeat}(n) = n : \text{repeat}(n) \quad \text{//n:n:n:n:...} \]
\[ \text{one_two}() = 1 : 2 : \text{one_two}() \quad \text{//1:2:1:2...} \]

Note: \([\cdot][\cdot][\cdot]\) are pointwise operations on streams, \(\text{\_}\) computes the tail.

Non-regular streams
\[ \text{nat}() = 0 : (\text{nat}()[+]\text{repeat}(1)) \]
\[ \text{fib}() = 0 : 1 : (\text{fib}()[+]\text{fib}()^{-}) \]

\[ \begin{align*}
\text{nat}() & \rightarrow \begin{array}{cccc}
1 & 1 & 1 & 1
\end{array} \\
\text{fib}() & \rightarrow \begin{array}{cccc}
1 & 2 & 3 & 5
\end{array}
\end{align*} \]

Functions for stream processing
\[ \text{aggr}(n, s) = \text{if } n < 0 \text{ then } \text{repeat}(0) \]
\[ \quad \text{else } s[+]\text{aggr}(n-1, s^-) \]
\[ \text{avg}(n, s) = \text{aggr}(n, s)[/]\text{repeat}(n) \]

Below you find an example of execution of \(\text{avg}\) over a window of size 3:
\[ \begin{array}{cc}
\hline
1 & 2 & 3 & 4 & 5 \\
\hline
\text{avg} & = & 4 & 5 & 6
\end{array} \]

Checking equality of streams
• The stream of all ones \(s = 1 : s\) is equal to its tail:
  \[ s = s^- \rightarrow s = (1 : s^-) \rightarrow s = s \]
• If we add more ones to stream \(s\) we still get \(s\) as a result:
  \[ 1 : 1 : s = 1 : s \rightarrow 1 : s = s \rightarrow 1 : s = 1 : s \rightarrow 1 : s = s = s \]

Forthcoming Research
• Make function definitions more \textit{flexible}
  The user is allowed to specify the behaviour in presence of a cycle
• Introduce a static type system to prevent runtime errors

Reference papers:

Davide Ancona, Pietro Barbieri, Elena Zucca. Enhanced Regular Corecursion for Data Streams. ICTCS ’21
Davide Ancona, Pietro Barbieri, Elena Zucca. Enhancing Expressivity of Checked Corecursive Streams. FLOPS ’22

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