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Vector Graphics on Surfaces Using Straightedge and Compass Constructions

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Introduction

The ancient Greek mathematicians developed a set of geometric techniques, which go under the name of *straightedge and compass constructions*, to draw a number of planar geometric figures and arrangements, involving straight lines, circles, and angles. The straightedge and compass constructions can be used to define vector graphics in the plane. In fact, several graphics primitives and constructions made available in the GUI of drawing systems can be addressed with such basic tools. This work is part of our effort to bring vector graphics to the manifold domain, i.e., by assuming a surface as a canvas [1, 2]. Here, we investigate to which extent the straightedge and compass constructions can be ported to the manifold setting, by using equivalent tools.

The main challenge is that constructions in the Euclidean context rely on geometric properties that no longer hold in the geodesic metric. In fact, even the basic properties of straight lines and circles do not hold on a surface without additional conditions. For instance, a long-enough geodesic line may self-intersect; there might exist infinitely many geodesic segments of different lengths joining two distinct points; and even the shortest geodesic segment between a pair of points might not be unique. Similar issues arise for circles: a generic isoline of the distance field from a point is guaranteed neither to be homeomorphic to a standard circle, nor to be smooth at all points; and equal angles at the center do not intercept equal chords or arcs on the circle. Figure 1 shows some example of such configurations.

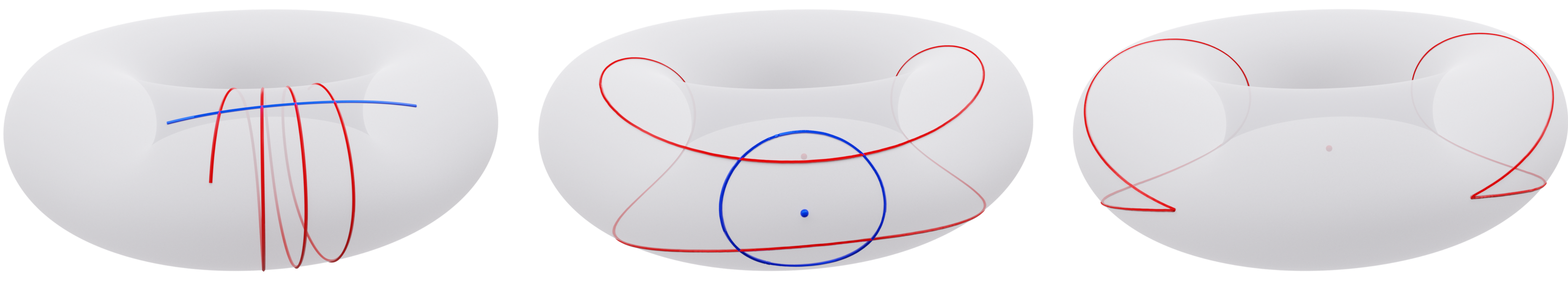

Figure 1: The intersection between two geodesics or two circles could be not topologically consistent with what happens in the Euclidean plane: geodesics may intersect in more than one point (*left*) and circles may intersect in more than two points (*middle*); a circle with radius greater than the injectivity radius of its center may not be even homeomorphic to a Euclidean circle, and may be not smooth at the cut locus of the center (*right*).

Constructions in Tangent Space

One way of porting such straightedge and compass constructions is as follows: given an initial configuration of points on S, we use the log map centered at a suited point $c \in S$ to map such points onto the tangent space T_cS. We then apply the Euclidean construction in T_cS, and finally map the result onto S through $\exp_c$.

For example, given two points $c, v_1 \in S$, a "regular" geodesic polygon centered at c and having one of its vertices at v_1 can be constructed in the following way. We lift point v_1 to point $\bar{v}_1$ in the tangent plane T_cS. Next, we apply the proper construction in tangent space, as in Figure 2; then we map the vertices of the resulting polygon to S with the exp map; and finally we connect them with geodesic lines.

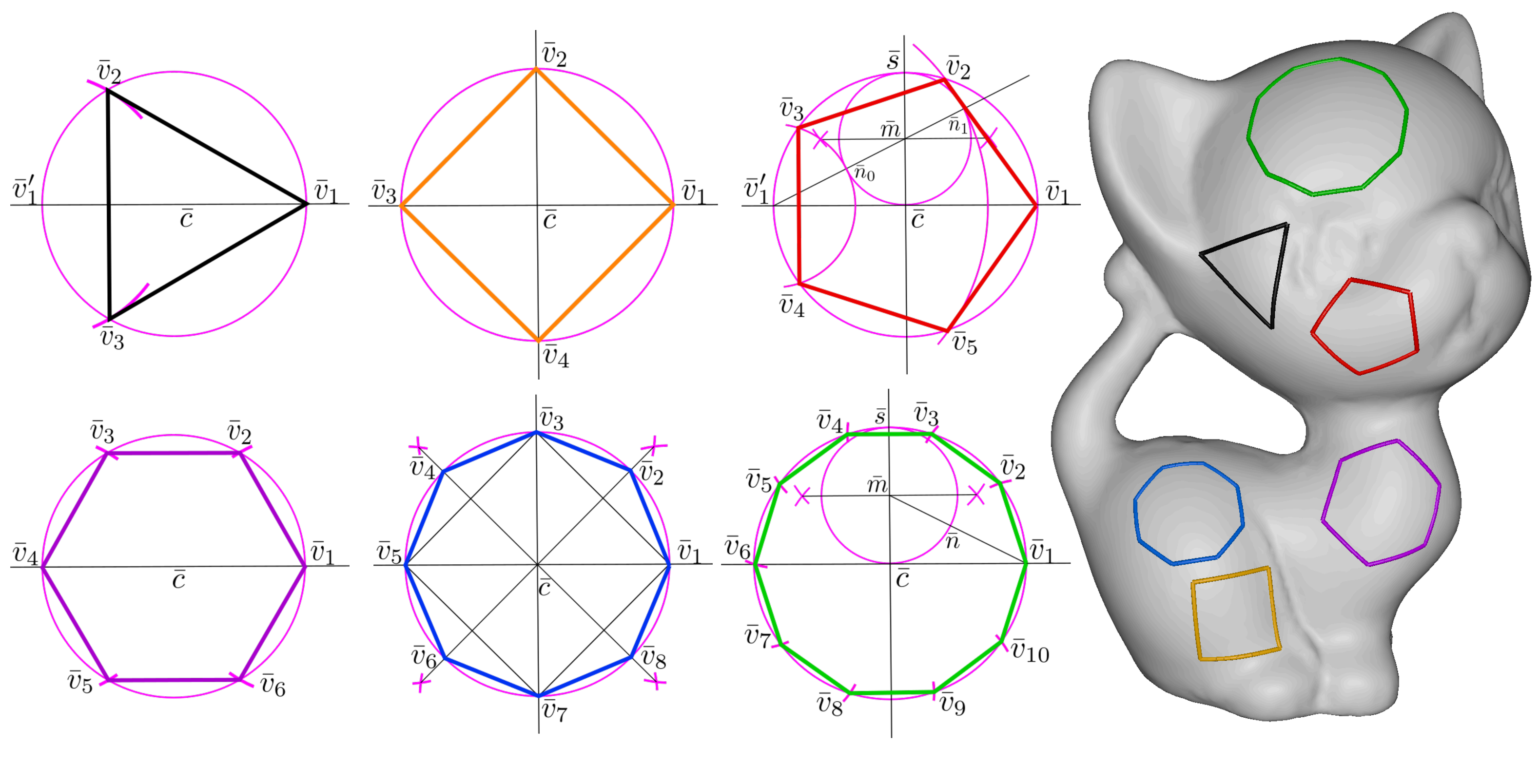

Figure 2: Euclidean construction of an inscribed regular n-gon for $n = 3, 4, 5, 6, 8, 10$ (*left*) and the results obtained by mapping such constructions on a mesh (*right*).

Direct Constructions on the Surface

Another approach consists in defining the manifold counterparts of the five basic constructions used in the Euclidean context:

1. line through two existing points;
2. circle through one point with center another point;
3. intersection point of two non-parallel lines;
4. intersection points of a line and a circle;
5. intersection points of two circles.

In this way we can perform constructions satisfying properties that we were not able to ensure in the previous case. For example, we have more leeway in defining a geodesic rectangle in terms of its angles and the length of its sides, as summarized in Figure 3.

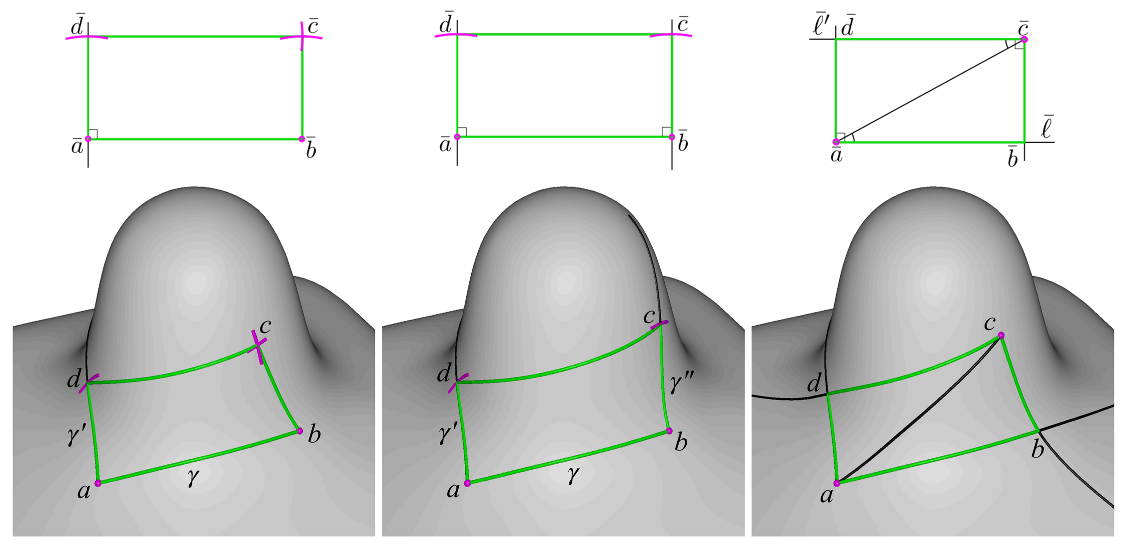

Figure 3: Rectangles obtained with different constructions: by tracing two perpendicular lines γ and γ' intersecting at a and tracing opposite sides of the same length (*left*); by tracing two lines γ' and γ'' perpendicular to γ at a and b and setting points d and c on γ' and γ'' at equal distance from a and b, respectively (*center*); by tracing the diagonal ac, transferring angle bac to acd and tracing two lines perpendicular to ab and cd at a and c, respectively (*right*).

Drawing System

We have developed a prototype system that supports their interactive usage on meshes up to the size of millions of triangles. We provide some macro-operations, which combine different primitive constructions to obtain complex decorations at once. Some examples of macros are shown in the decorations in Figure 4.

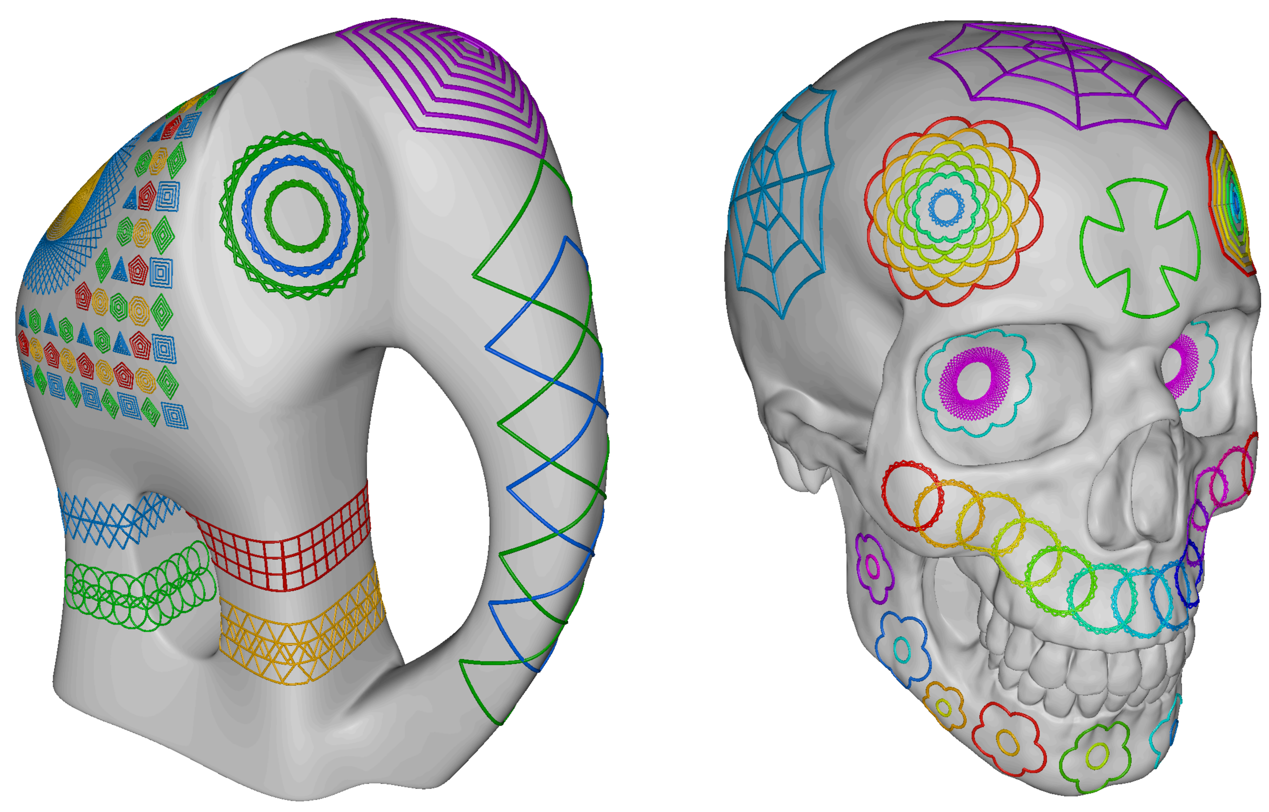

Figure 4: Examples of drawings obtained interactively with our prototype system on two meshes, each consisting of 1M triangles.

References

[1] Claudio Mancinelli, Giacomo Nazzaro, Fabio Pellacini, and Enrico Puppo. b/Surf: Interactive Bézier Splines on Surface Meshes. *IEEE Transactions on Visualization and Computer Graphics*, 2022.

[2] Giacomo Nazzaro, Enrico. Puppo, and Fabio Pellacini. *geoTangle:* Interactive Design of Geodesic Tangle Patterns on Surfaces. *ACM Trans. Graph.*, 41(2):12:1–12:17, 2022.

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