Università di Genova
DIPARTIMENTO INFORMATICA, BIOINGEGNERIA, ROBOTICA E INGEGNERIA DEI SISTEMI

Computer Science Workshop
PhD program in Computer Science and Systems Engineering

Learning-Based Low-Rank Denoising: An application to biomedical images

Simone Cammarasana, Giuseppe Patanè

Introduction

Ultrasound images are widespread in medical diagnosis for muscle-skeletal, cardiac, and obstetrical diseases, due to the efficiency and non-invasiveness of the acquisition methodology. Ultrasound acquisition is affected by speckle noise, which corrupts the resulting image and affects further processing steps.

In this context, the denoising of ultrasound images is relevant both for post-processing and visual evaluation by the physician [1].

Main Objectives

1. Denoising of 2D images through low-rank methods;
2. Learning and prediction of the optimal thresholds of the Singular Values Decomposition (SVD);
3. Specialisation to different noise types and intensities;
4. Application to ultrasound 2D images.

Materials and Methods

Given a training data set of ground-truth images, we extract 3D blocks through the block matching algorithm [2], and compute the optimal singular values through a proper optimisation applied to the SVD of each 3D block. We iterate this approach, where the input image of each iteration is the denoised image at the previous step.

The input and optimal singular values compose the training data set of the learning model; the singular values of the training data set are classified according to four parameters: noise type and intensity, number of iterations of the method, and clustering of the singular values. This classification allows us to design specific networks and improve the accuracy of the learning model. The learning phase optimises a matrix of weights, applied to train the model and predict the optimal thresholds of the SVD.

For the image denoising, we compute the 3D blocks and the SVD, the four parameters of each 3D block, and the thresholds through the network's prediction. Finally, the block-matching aggregation is applied to reconstruct the denoised image.

Mathematical Section

An input image $\mathbf{Y} = f(\mathbf{X}, \mathbf{N})$ is composed of the ground-truth $\mathbf{X}$ and the noisy component $\mathbf{N}$, where f defines the combination of one or more type of noise (e.g., additive, impulsive, multiplicative). Low-rank methods [3] recover the approximated image $\hat{\mathbf{X}} = \mathbf{U}(\tau(\mathbf{S}))\mathbf{V}^\intercal$, where $\mathbf{Y} = \mathbf{USV}^\intercal$, $\mathbf{S} := \text{diag}(\mathbf{s})$, is the SVD of the noisy signal, and $\tau(\mathbf{S}) = \text{diag}(\mathbf{S}_{hh} - \lambda_h)$, where $\mathbf{S}_{hh}$ is the (h,h) entry of $\mathbf{S}$. The proposed learning-based method trains a network to predict the optimal thresholds λ_h.

Given a $n \times n$ ground truth image $\mathbf{X}$ and a noisy instance $\mathbf{Y} = f(\mathbf{X}, \mathbf{N})$, we compute the optimal thresholds λ_h through the minimisation of the distance between the ground-truth and the reconstructed signal $\hat{\mathbf{X}}$, as

$$E(\lambda) = \sum_{i=1}^{n}\sum_{j=1}^{n} |\mathbf{X}_{ij} - \hat{\mathbf{X}}_{ij}|^2,$$

where the reconstructed signal is computed through the SVD of the $\mathbf{Y}$ signal with weighted thresholds, i.e., $\hat{\mathbf{X}}_{ij} = \sum_{h=1}^{n} \mathbf{U}_{ih}(\mathbf{S}_{hh} - \lambda_h)\mathbf{V}_{hj}$.

Finally, we train a network to predict the thresholds that reconstruct the best approximation of each ground-truth block. The network weights are defined through the matrix $\mathbf{W}$ of dimension $n \times n$, and are computed by minimising the distance between the target and the predicted thresholds, as

$$F(\mathbf{W}) = \sum_{k=1}^{N}\sum_{j=1}^{n} \left|\mathbf{Q}_{jk} - \hat{\mathbf{Q}}_{jk}\right|^2,$$

where $\hat{\mathbf{Q}}_{jk} = \sum_{i=1}^{n} \mathbf{W}_{ij}\mathbf{P}_{ik}$ are the predicted thresholds.

Experimental results

Fig. 1 shows the results of our denoising method, with different anatomical districts. Fig. 2 shows the prediction of the optimal thresholds.

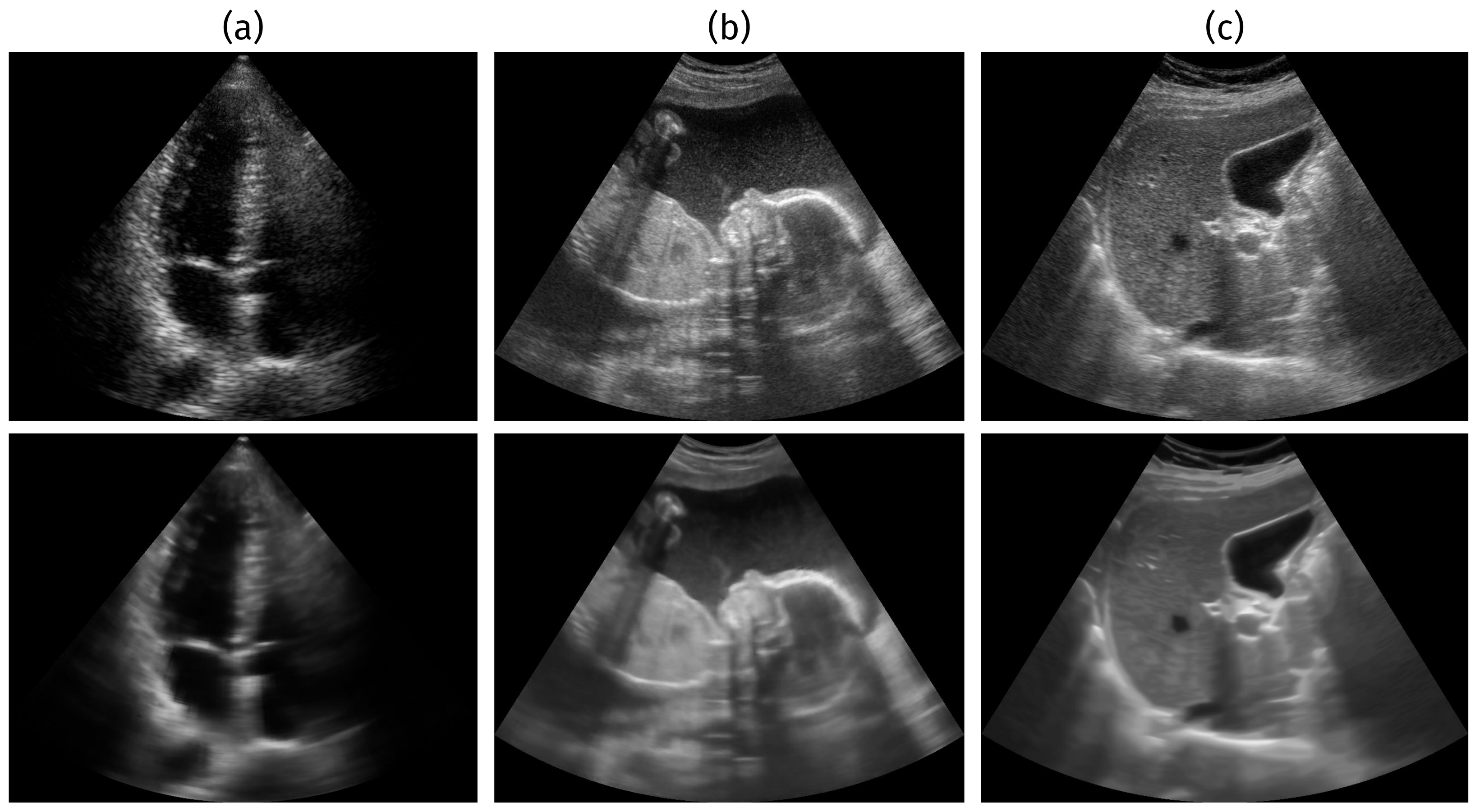

Figure 1: Raw (first row) and denoised (second row): (a) cardiac district; (b) obstetrical distrit; (c) abdominal district.

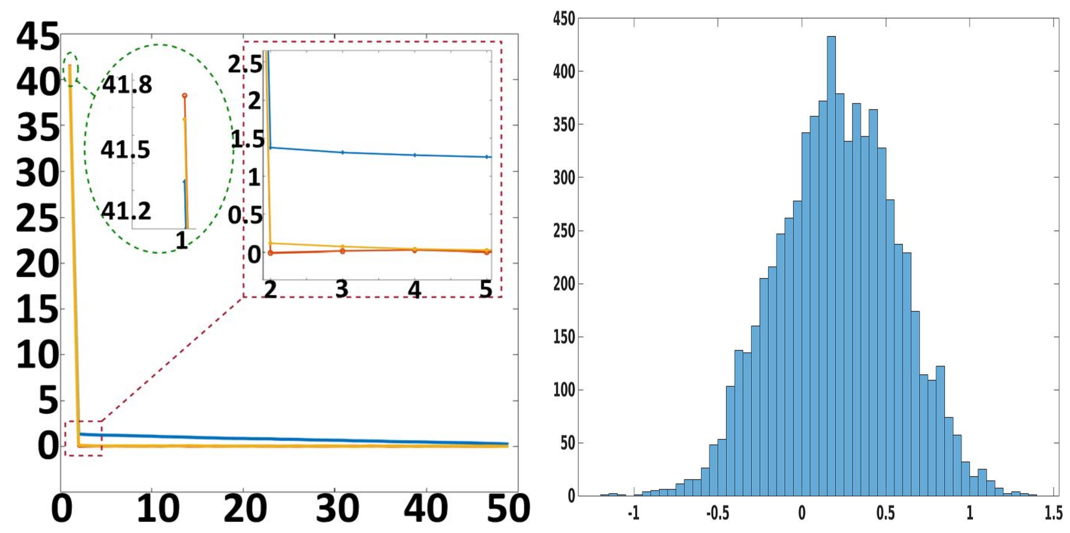

Figure 2: Left: Input (blue), optimal (orange), and predicted (yellow) SV. Right: error on SV prediction.

Conclusions

We propose a novel denoising method, with an application to 2D ultrasound images, from different anatomical districts. Our method exploits the SVD image factorisation, and a learning model of the optimal thresholds.

The proposed framework is general with respect to input signals (2D images, videos, graphs), signal transformation (SVD, Wavelet, Dictionary learning), and the applications (denoising, clustering, super-resolution).

Forthcoming Research

We plan to apply our denoising method to signals acquired with different methodologies (e.g., 3D ultrasound, MRI), also taking into account time-dependent data (e.g., ultrasound videos). Finally, the industrial and clinical validations of the proposed framework are under development, by comparing our results with tools currently used in medical clinics.

References

[1] Simone Cammarasana, Paolo Nicolardi, and Giuseppe Patanè. A universal deep learning framework for real-time denoising of ultrasound images. *arXiv preprint arXiv:2101.09122*, 2021.

[2] Kostadin Dabov, Alessandro Foi, Vladimir Katkovnik, and Karen Egiazarian. Image denoising with block-matching and 3D filtering. In *Image Processing: Algorithms and Systems, Neural Networks, and Machine Learning*, volume 6064, page 606414, 2006.

[3] Shuhang Gu, Qi Xie, Deyu Meng, Wangmeng Zuo, Xiangchu Feng, and Lei Zhang. Weighted nuclear norm minimization and its applications to low level vision. *Inter. Journal of Computer Vision*, 121(2):183–208, 2017.

Acknowledgements

This research is carried out as part of an Industrial PhD project funded by CNR-IMATI and Esaote S.p.A. under the CNR-Confindustria agreement.

CONTACTS

Simone Cammarasana
simone.cammarasana@ge.imati.cnr.it

Giuseppe Patanè
patane@ge.imati.cnr.it